DEPARTMENT OF MATHEMATICAL SCIENCES

UNIVERSITY OF GOTHENBURG

NFMV017   Calculus of Variations, 7.5 credits

Variationskalkyl, 7,5 högskolepoäng

Third-cycle level / Forskarnivå

Confirmation

This syllabus was confirmed by the Department of Mathematical Sciences on 2021-04-29, and was last revised on 2022-06-23. The revised course syllabus is valid from Autumn semester 2022.

Responsible Department
Department of Mathematical Sciences, Faculty of Science

Entry requirements

Several variables calculus, Sobolev spaces, basic knowledge in partial differential equations and functional analysis. Some basic tools (e.g., the notion of weak convergence in Sobolev spaces) will be reviewed in the course

Learning outcomes

At the end of the course the students will be able to:

- Derive the Euler-Lagrange equations for variational problems with constraints
- Prove the existence of minimizers to convex functionals subject to different types of constraints
- Proving existence of (weak) solutions to certain PDE's by variational methods

Course content

Part I. Fundamental topics:

Variational problems and Euler-Lagranger equations

Existence, uniqueness and regularity of minimizers of coercive convex functionals

Variational problems with constraints

Systems of Euler-Lagrange equations

Critical points and Mountain pass theorem

Part II. Advanced topics:

Minimizers of the energy functional of elastic bodies

Polyconvexity

Injective minimizers

Concentration-Compactness Lemma

Symmetric decreasing rearrangements

Minimizers of the energy functional of self-gravitating fluids

Types of instruction

Lectures 2 x 2h per week.

The first part (basic topics) will last 4 weeks and will be based mostly on Chapter 8 in L. C. Evans, *Partial Differential Equations* (AMS)

The second part (advanced topics) will last 3 weeks and will be based mostly on original papers, plus some material from E. H. Lieb, M. Loss, *Analysis* (AMS)

Language of instruction
The course is given in English.

Grades

The grade Pass (G) or Fail (U) is given in this course.

Types of assessment

Homework exercises and oral presentation of the solutions to the examiner

Course evaluation

Together with two students, we will evaluate the course at the end

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