UNIVERSITY OF GOTHENBURG

DEPARTMENT OF MATHEMATICAL SCIENCES

NFMV025 Canonical metrics on Kähler manifolds and holomorphic vector bundles, 7.5 credits

Kanoniska metriker på Kählermångfalder och holomorfa vektorknippen, 7,5 högskolepoäng

Third-cycle level / Forskarnivå

Confirmation

This syllabus was confirmed by the Department of Mathematical Sciences on 2022-09-02, and is valid from Autumn semester 2022.

Responsible Department
Department of Mathematical Sciences, Faculty of Science

Entry requirements

Some basic differential geometry and complex geometry. Some experience with (elliptic) PDEs preferable, but we will go over the key points needed for our purposes in the lectures. Some parts of the course will feature some algebraic geometry, but this will be more focused on the ideas rather than precise proofs, so no real background with algebraic geometry is necessary.

Learning outcomes

The course is an introduction to canonical metrics in complex geometry, both on Kähler manifolds and on holomorphic vector bundles over them. At the end of the course, students will be able to understand various constructions of such metrics as well as obstructions to their existence coming from algebraic geometry. More precisely, students will be able to understand:

- Some constructions of canonical Kähler metrics (such as on Riemann surfaces and on canonically polarised and Calabi-Yau manifolds).
- That there are obstructions to the existence of canonical Kähler metrics coming from algebraic geometry.
- The main points of the Hitchin-Kobayashi correspondence between existence of Hermite-Einstein metrics on vector bundles and slope stability.

Course content

- The Hodge decomposition theorem on Kähler manifolds
- The Uniformisation theorem
- The existence of Kähler-Einstein metrics on canonically polarised/Calabi-Yau manifolds
- The classical Futaki invariant as an obstruction to the existence of constant scalar curvature Kähler (cscK) metrics
- The LeBrun-Simanca openness theorem for cscK/extremal metrics
- The infinite dimensional moment map picture for the scalar curvature and K-stability as an obstruction to the existence of cscK metrics
- The Hitchin-Kobayashi correspondence for holomorphic vector bundles

Types of instruction

2h lecture per week for about 2 months. This will be followed by student presentations on papers, which forms the main part of the assessment of the course.

Language of instruction
The course is given in English.

Grades

The grade Pass (G) or Fail (U) is given in this course.

Types of assessment

Some homework exercises during the course. Oral presentation of a paper/topic related to the content of the course at the end of the course. Students can either ask the instructor for suggestion on what paper to talk about or come with their own suggestion, which then has to be approved.

Course evaluation

We will evaluate the course together with two students after the end of the lectures.

Other information

For the part of the course on canonical Kähler metrics, we will follow Gábor Székelyhidi's book "Introduction to Extremal Kähler Metrics". The preliminary version of the book is available online at https://www3.nd.edu/~gszekely/notes.pdf. Other useful sources (at least for some parts of the course) include Daniel Huybrechts's "Complex geometry - an introduction" and Simon Donaldson's "Riemann Surfaces". For the part of the course on vector bundles, we will follow Shoshichi Kobayashi's "Differential geometry of complex vector bundles". I will also aim to type up lecture notes that will be available on my personal home page.